Excited States of the Nucleon in 2+1 flavour QCD

Derek Leinweber CSSM Lattice Collaboration

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Variational Method Two-Point Correlation Functions Lattice Simulation Results Interpolating Fields Discovering More States Formalism Summary of Results Eigenstate-Projected Correlator

• Two point correlation function:

$$G_{ij}(t, \vec{p}) = \sum_{\vec{x}} e^{-i\vec{p}.\vec{x}} \langle \Omega | T\{\chi_i(x)\bar{\chi}_j(0)\} | \Omega \rangle.$$

Inserting completeness

$$\sum_{B,ec{p'},s} |B,ec{p'},s
angle\langle B,ec{p'},s|=I$$

Then

$$\begin{aligned} \mathbf{G}_{ij}(t,\vec{p}) &= \sum_{B^+} \lambda_{B^+} \bar{\lambda}_{B^+} \mathbf{e}^{-\mathbf{E}_{B^+}t} \frac{\gamma \cdot \mathbf{p}_{B^+} + M_{B^+}}{2\mathbf{E}_{B^+}} \\ &+ \sum_{B^-} \lambda_{B^-} \bar{\lambda}_{B^-} \mathbf{e}^{-\mathbf{E}_{B^-}t} \frac{\gamma \cdot \mathbf{p}_{B^-} - M_{B^-}}{2\mathbf{E}_{B^-}} \end{aligned}$$

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Two-Point Correlation Functions Interpolating Fields Formalism Eigenstate-Projected Correlators

• At
$$\vec{p} = 0$$

$$egin{aligned} G^{\pm}_{ij}(t,ec{0}) &= \mathrm{Tr}_{\mathrm{sp}}[\Gamma_{\pm}G_{ij}(t,ec{0})] \ &= \sum_{B^{\pm}}\lambda^{\pm}_{i}ar{\lambda}^{\pm}_{j}\mathbf{e}^{-M_{B^{\pm}}t}. \end{aligned}$$

• Parity projection operator,

$$\Gamma_{\pm}=\frac{1}{2}(1\pm\gamma_0).$$

Asymptotically

$$G_{ij}^{\pm}(t,\vec{0}) \stackrel{t\to\infty}{=} \lambda_{i0}^{\pm}\bar{\lambda}_{j0}^{\pm}e^{-M_{0^{\pm}}t}.$$

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Two-Point Correlation Functions Interpolating Fields Formalism Eigenstate-Projected Correlators

Interpolators

Consider

$$egin{aligned} \chi_1(x) &= \epsilon^{abc}(u^{Ta}(x)\,C\gamma_5\,d^b(x))\,u^c(x)\,, \ \chi_2(x) &= \epsilon^{abc}(u^{Ta}(x)\,C\,d^b(x))\,\gamma_5\,u^c(x)\,, \ \chi_4(x) &= \epsilon^{abc}(u^{Ta}(x)\,C\gamma_5\gamma_4\,d^b(x))\,u^c(x). \end{aligned}$$



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Variational Method

• Consider N interpolating fields, then

$$\bar{\phi}^{\alpha} = \sum_{i=1}^{N} u_i^{\alpha} \bar{\chi}_i,$$
$$\phi^{\alpha} = \sum_{i=1}^{N} v_i^{\alpha} \chi_i,$$

such that,

$$\langle \boldsymbol{B}_{\!\beta}, \boldsymbol{p}, \boldsymbol{s} | ar{\phi}^{lpha} | \Omega
angle = \delta_{lpha eta} ar{m{z}}^{lpha} ar{m{u}}(lpha, m{p}, m{s}),$$

$$\langle \Omega | \phi^{\alpha} | B_{\beta}, p, s \rangle = \delta_{\alpha\beta} z^{\alpha} u(\alpha, p, s),$$

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Two-Point Correlation Functions Interpolating Fields Formalism Eigenstate-Projected Correlators

• Then a two point correlation function matrix for $\vec{p} = 0$, right multiplied by u_i^{α} has the property

$$egin{aligned} G^{\pm}_{ij}(t) \, u^{lpha}_{j} &= (\sum_{ec{\mathbf{X}}} \mathrm{Tr}_{\mathrm{sp}}\{ \mathsf{\Gamma}_{\pm} \langle \Omega | \chi_i ar{\chi}_j | \Omega
angle \}) \, u^{lpha}_{j} \ &= \lambda^{lpha}_i ar{\mathbf{Z}}^{lpha} \mathbf{e}^{-m_{lpha} t}. \end{aligned}$$

(no sum over α)

• The *t* dependence is contained in the exponential term

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• This provides a recurrence relation at time $(t_0 + \triangle t)$,

$$G_{ij}(t_0 + \bigtriangleup t)u_j^{lpha} = e^{-m_{lpha}\bigtriangleup t}G_{ij}(t_0)u_j^{lpha}.$$

• Multiplying by $[G_{ij}(t_0)]^{-1}$ from left,

$$[(\boldsymbol{G}(t_0))^{-1}\boldsymbol{G}(t_0+\bigtriangleup t)]_{ij}\,\boldsymbol{u}_j^{\alpha}=\boldsymbol{c}^{\alpha}\,\boldsymbol{u}_j^{\alpha},$$

- where $c^{\alpha} = e^{-m_{\alpha} \Delta t}$ is the eigenvalue.
- Similarly, it can also be solved for the left eigenvalue equation for ν^α eigenvector,

$$\mathsf{v}_i^lpha\,[\mathsf{G}(\mathit{t}_0+ riangle t)(\mathsf{G}(\mathit{t}_0))^{-1}]_{ij}=\mathsf{c}^lpha\,\mathsf{v}_j^lpha.$$

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• The vectors u_j^{α} and v_i^{α} diagonalize the correlation matrix at time t_0 and $t_0 + \triangle t$ making the projected correlation function

$$v_i^{lpha} G_{ij}(t) u_j^{eta} = \delta^{lphaeta} z^{lpha} ar{z}^{eta} \mathbf{e}^{-m_{lpha}t}$$

 The projected correlator, is then analyzed to obtain masses of different states,

$$v_i^{lpha}G_{ij}^{\pm}(t)u_j^{lpha}\equiv G_{\pm}^{lpha},$$

• Our effective mass is defined as

$$M^lpha_{
m eff}(t) = \ln\left(rac{G^lpha_\pm(t,ec{0})}{G^lpha_\pm(t+1,ec{0})}
ight).$$

 Variational Method
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 Summary of Results
 Eigenstate-Projected Correlators

4 \times 4 correlation matrix of χ_1 with 4 smearing levels

Projected Mass



Mass From Eigenvalue



• $t_{\text{start}} = t_0$ is shown in major tick marks

• $\triangle t$ is shown in minor tick marks

PACS-CS Simulation Details Source/Sink Smearing Method Roper in Dynamical-Fermion QCD N1/2⁻ State in Dynamical-Fermion QCD

PACS-CS Simulation Details

PACS-CS Collaboration: S. Aoki, et al., Phys. Rev. **D79** (2009) 034503.

- Lattice volume: $32^3 \times 64$
- Non-perturbative O(a)-improved Wilson quark action
- Iwasaki gauge action
- 2 + 1 flavour dynamical-fermion QCD
- $\beta = 1.9$ providing a = 0.0907 fm
- $K_{ud} = \{ 0.13700, 0.13727, 0.13754, 0.13770, 0.13781 \}$
- $K_{\rm s} = 0.13640$
- Lightest pion mass is 156 MeV.

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PACS-CS Simulation Details Source/Sink Smearing Method Roper in Dynamical-Fermion QCD N1/2⁻ State in Dynamical-Fermion QCD

Sommer Scale

Lattice spacing is set via the force between static quarks

$$\left. r_{c}^{2} \left. \frac{\partial V(r)}{\partial r} \right|_{r=r_{c}} = c \right.$$

- Sommer prefers c = 1.65, such that $r_c = r_0 = 0.49$ fm
- The Sommer scale facilitates comparisons with other results

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PACS-CS Simulation Details Source/Sink Smearing Method Roper in Dynamical-Fermion QCD N1/2⁻ State in Dynamical-Fermion QCD

Source Smearing

Correlation matrices are built from a variety of source and sink smearings.

$$\psi_i(\mathbf{x},t) = \sum_{\mathbf{x}'} F(\mathbf{x},\mathbf{x}') \psi_{i-1}(\mathbf{x}',t),$$

where,

$$\begin{aligned} F(\boldsymbol{x}, \boldsymbol{x}') &= (1 - \alpha) \delta_{\boldsymbol{x}, \boldsymbol{x}'} + \frac{\alpha}{6} \sum_{\mu=1}^{3} [U_{\mu}(\boldsymbol{x}) \delta_{\boldsymbol{x}', \boldsymbol{x} + \hat{\mu}} \\ &+ U_{\mu}^{\dagger}(\boldsymbol{x} - \hat{\mu}) \delta_{\boldsymbol{x}', \boldsymbol{x} - \hat{\mu}}], \end{aligned}$$

Fixing $\alpha = 0.7$, the procedure is repeated $N_{\rm sm}$ times.

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PACS-CS Simulation Details Source/Sink Smearing Method Roper in Dynamical-Fermion QCD N1/2⁻ State in Dynamical-Fermion QCD

Smeared Source - Point Sink Effective Masses

For second lightest quark mass and 50 configurations



PACS-CS Simulation Details Source/Sink Smearing Method Roper in Dynamical-Fermion QCD N1/2⁻ State in Dynamical-Fermion QCD

4 \times 4 bases of $\chi_1 \bar{\chi}_1$

Sweeps \rightarrow	16	25	35	50	70	100	125	200	400	800		
Basis No. \downarrow	Bases											
1	16	-	35	-	70	100	-	-	-	-		
2	16	-	35	-	70	-	125	-	-	-		
3	16	-	35	-	-	100	-	200	-	-		
4	16	-	35	-	-	100	-	-	400	-		
5	16	-	-	50	-	100	125	-	-	-		
6	16	-	-	50	-	100	-	200	-	-		
7	16	-	-	50	-	-	125	-	-	800		
8	-	25	-	50	-	100	-	200	-	-		
9	-	25	-	50	-	100	-	-	400	-		
10	-	-	35	-	70	-	125	-	400	-		

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All 4×4 bases: second lightest mass



PACS-CS Simulation Details Source/Sink Smearing Method Roper in Dynamical-Fermion QCD N1/2⁻ State in Dynamical-Fermion QCI

Even Parity Nucleon Spectrum in full QCD



PACS-CS Simulation Details Source/Sink Smearing Method Roper in Dynamical-Fermion QCD N1/2⁻ State in Dynamical-Fermion QCI

Even Parity Nucleon Spectrum in full QCD



PACS-CS Simulation Details Source/Sink Smearing Method Roper in Dynamical-Fermion QCD N1/2⁻ State in Dynamical-Fermion Q

Even Parity Nucleon Spectrum in Quenched QCD



PACS-CS Simulation Details Source/Sink Smearing Method Roper in Dynamical-Fermion QCD N1/2⁻ State in Dynamical-Fermion QCI

Even Parity Nucleon Spectrum in full QCD



PACS-CS Simulation Details Source/Sink Smearing Method Roper in Dynamical-Fermion QCD N1/2⁻ State in Dynamical-Fermion QCI

Even Parity Nucleon Spectrum in full QCD



PACS-CS Simulation Details Source/Sink Smearing Method Roper in Dynamical-Fermion QCD N1/2⁻ State in Dynamical-Fermion QCI

Even Parity Nucleon Spectrum in full QCD



PACS-CS Simulation Details Source/Sink Smearing Method Roper in Dynamical-Fermion QCD N1/2⁻ State in Dynamical-Fermion QCD

$N_{\frac{1}{2}}^{1-}$ (1535) State



PACS-CS Simulation Details Source/Sink Smearing Method Roper in Dynamical-Fermion QCD N1/2⁻ State in Dynamical-Fermion QCD

$N_{\frac{1}{2}}^{1-}$ (1535) State



PACS-CS Simulation Details Source/Sink Smearing Method Roper in Dynamical-Fermion QCD N1/2⁻ State in Dynamical-Fermion QCD

$N_{\frac{1}{2}}^{1-}$ (1535) State



PACS-CS Simulation Details Source/Sink Smearing Method Roper in Dynamical-Fermion QCD N1/2⁻ State in Dynamical-Fermion QCD

$N_{\frac{1}{2}}^{1-}$ (1535) State



PACS-CS Simulation Details Source/Sink Smearing Method Roper in Dynamical-Fermion QCD N1/2⁻ State in Dynamical-Fermion QCD

Loss of multi-particle states at light quark masses

Consider an interpolator creating a resonance with a finite width in the infinite volume limit.

On a finite volume lattice:

- The energy thresholds of the states involve discretised momenta n(2π/L).
- The density of states increases with the lattice volume $V = L^3$.
- The coupling to the meson-baryon states is suppressed by $1/\sqrt{V}$.
- This provides a finite width and finite spectral strength as $V \to \infty$.
- The strength of the volume suppression effect depends on the resonance width.
- The width of resonances with heavy quarks are relatively small.

PACS-CS Simulation Details Source/Sink Smearing Method Roper in Dynamical-Fermion QCD N1/2⁻ State in Dynamical-Fermion QCD

 $N_{\frac{1}{2}}^{1+}$ (940), $N_{\frac{1}{2}}^{1+}$ (1440), $N_{\frac{1}{2}}^{1-}$ (1535) States



PACS-CS Simulation Details Source/Sink Smearing Method Roper in Dynamical-Fermion QCD N1/2⁻ State in Dynamical-Fermion QCD

Roper and $N1/2^{-}$ states in Quenched QCD



Mahbub et al., Phys. Lett. B 693, 351 (2010), [arXiv:hep-lat/1007.4871].

PACS-CS Simulation Details Source/Sink Smearing Method Roper in Dynamical-Fermion QCD N1/2⁻ State in Dynamical-Fermion QCD

Quenched Vs Dynamical, N^+ states



PACS-CS Simulation Details Source/Sink Smearing Method Roper in Dynamical-Fermion QCD N1/2⁻ State in Dynamical-Fermion QCD

Quenched Vs Dynamical, N^+ states



PACS-CS Simulation Details Source/Sink Smearing Method Roper in Dynamical-Fermion QCD N1/2⁻ State in Dynamical-Fermion QCD

Quenched Vs Dynamical, N^+ states



PACS-CS Simulation Details Source/Sink Smearing Method Roper in Dynamical-Fermion QCD N1/2⁻ State in Dynamical-Fermion QCD

Quenched Vs Dynamical, N^- states



Variational MethodExpanding the Correlation Matrix via χ_1, χ_2, χ_4 Lattice Simulation Results N^- SpectrumDiscovering More States N^- SpectrumSummary of Results Λ^- Spectrum

N^+ Spectrum for heaviest m_q : 4 × 4 \rightarrow 8 × 8 $\chi_1 \chi_2$



 Variational Method
 Expanding the Correlation Matrix via χ_1, χ_2, χ_4

 Lattice Simulation Results
 N^+ Spectrum

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 Λ^- Spectrum

N^+ Spectrum for heaviest m_q : 8 × 8 $\chi_2 \chi_4$



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N^+ Spectrum for heaviest m_q : 8 × 8 $\chi_1 \chi_4$



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N^+ Spectrum for heaviest m_q : 12 × 12 $\chi_1 \chi_2 \chi_4$



 Variational Method
 Expanding the Correlation Matrix via χ_1, χ_2, χ_4

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 N^+ Spectrum

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 N^- Spectrum

N^+ Spectrum: P-wave $N\pi$ threshold



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Variational MethodExpanding the Correlation Matrix via χ_1, χ_2, χ_4 Lattice Simulation Results \mathcal{N}^{-} SpectrumDiscovering More States \mathcal{N}^{-} SpectrumSummary of Results \mathcal{N}^{-} Spectrum

N⁺ Spectrum for 2nd heaviest m_q : 4 × 4 → 8 × 8 $\chi_1 \chi_2$



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N^+ Spectrum for 2nd heaviest m_q : 8 × 8 $\chi_2 \chi_4$



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N^+ Spectrum for 2nd heaviest m_q : 8 × 8 $\chi_1 \chi_4$



Variational MethodExpanding the Correlation Matrix via χ_1, χ_2, χ_4 Lattice Simulation Results N^{-} SpectrumDiscovering More States N^{-} SpectrumSummary of Results Λ^{-} Spectrum

N^+ Spectrum for 2nd heaviest m_q : 12 × 12 $\chi_1 \chi_2 \chi_4$



Variational MethodExpanding the Correlation Matrix via χ_1, χ_2, χ_4 Lattice Simulation Results N^+ SpectrumDiscovering More States N^- SpectrumSummary of Results Λ^- Spectrum

N^+ Spectrum for 2nd heaviest m_q : HSC Comparison



Variational MethodExpanding the Correlation Matrix via χ_1, χ_2, χ_4 Lattice Simulation Results N^+ SpectrumDiscovering More States N^- SpectrumSummary of Results Λ^- Spectrum

N^+ Spectrum for 2nd heaviest m_q : HSC Rescaled



 Variational Method
 Expanding the Correlation Matrix via χ_1, χ_2, χ_4

 Lattice Simulation Results
 N^+ Spectrum

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 N^- Spectrum

N^+ Spectrum: P-wave $N\pi$ thresholds



N^+ Spectrum for 3rd m_q : 4 × 4 → 8 × 8 $\chi_1 \chi_2$



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Variational MethodExpanding the Correlation Matrix via χ_1, χ_2, χ_4 Lattice Simulation Results N^+ SpectrumDiscovering More States N^- SpectrumSummary of Results Λ^- Spectrum

N^+ Spectrum for 3rd m_q : 8 × 8 $\chi_2 \chi_4$



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 Variational Method
 Expanding the Correlation Matrix via χ_1, χ_2, χ_4

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N^+ Spectrum for 3rd m_q : 8 × 8 $\chi_1 \chi_4$



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N^+ Spectrum for 3rd m_q : 12 × 12 $\chi_1 \chi_2 \chi_4$



Expanding the Correlation Matrix via χ_1, χ_2, χ_4

- I⁺ Spectrum
- N⁻ Spectrum
- \" Spectrum

N^+ Spectrum for 3rd m_q : HSC Comparison



Expanding the Correlation Matrix via χ_1, χ_2, χ_4

- I⁺ Spectrum
- V⁻ Spectrum
- \" Spectrum

N^+ Spectrum for 3rd m_q : HSC Rescaled



 Variational Method
 Expanding the Correlation Matrix via χ_1, χ_2, χ_4

 Lattice Simulation Results
 N^+ Spectrum

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 N^- Spectrum

N^+ Spectrum: P-wave $N\pi$ thresholds



Expanding the Correlation Matrix via χ_1, χ_2, χ_4

- N⁺ Spectrum
- V⁻ Spectrum
- ∧[−] Spectrum

m_{π}^2 dependence of the N^+ Spectrum



 Variational Method
 Expanding the Correlation Matrix

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 N^- Spectrum

N^+ Spectrum: S and P-wave $N\pi$ thresholds



Expanding the Correlation Matrix via χ_1 , χ_2 , χ_4

N⁺ Spectrum

V⁻ Spectrum

∧[−] Spectrum

N⁺ Spectrum: HSC Comparison



 Variational Method
 Expanding the Correlation Matrix via χ_1, χ_2, χ_4

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Eigenvectors: Roper state at our lightest mass $m_{\pi} = 156 \text{ MeV}$



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N⁻ Spectrum for 2nd heaviest m_q : 4 × 4 \rightarrow 8 × 8 $\chi_1 \chi_2$



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Variational MethodExpanding the Correlation Matrix via χ_1, χ_2, χ_4 Lattice Simulation Results N^- SpectrumDiscovering More States N^- SpectrumSummary of Results Λ^- Spectrum

N^- Spectrum for 2nd heaviest m_q : HSC Comparison



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N^- Spectrum for 2nd heaviest m_q : HSC Rescaled



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N⁻ Spectrum for 3rd m_q : 4 × 4 → 8 × 8 $\chi_1 \chi_2$



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Expanding the Correlation Matrix via χ_1 , χ_2 , χ_4 N^+ Spectrum

N⁻ Spectrum

Spectrum

N^- Spectrum for 3rd m_q : HSC Comparison



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Expanding the Correlation Matrix via χ_1 , χ_2 , χ_4 N^+ Spectrum

N⁻ Spectrum

Spectrum

N^- Spectrum for 3rd m_q : HSC Rescaled



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Expanding the Correlation Matrix via χ_1 , χ_2 , χ_4 N^+ Spectrum

N⁻ Spectrum

\^ Spectrum

m_{π}^2 dependence of the N⁻ Spectrum



N^- Spectrum: S-wave $N\pi$ threshold



Variational Method Expanding the Correlation Matrix via χ_1, χ_2, \vdots Lattice Simulation Results N^+ Spectrum Summary of Results Λ^- Spectrum

N^- Spectrum: S and P-wave $N\pi$ thresholds



N^- Spectrum: S and P-wave $N\pi$ thresholds



Expanding the Correlation Matrix via χ_1 , χ_2 , χ_4 N^+ Spectrum

N⁻ Spectrum

∧[−] Spectrum

N⁻ Spectrum: HSC Comparison



Variational Method **Discovering More States**

 Λ^{-} Spectrum

Λ^- Spectrum: 6 × 6 $\chi_1 \chi_2$



Variational MethodExpanding the Correlation Matrix via χ_1, χ_2, χ_4 Lattice Simulation Results N^+ SpectrumDiscovering More States N^- SpectrumSummary of Results Λ^- Spectrum

Λ^- Spectrum: S-wave thresholds



Λ^- Spectrum: P-wave $NK\pi$ thresholds



 Variational Method
 Expanding the Correlation Matrix via χ_1, χ_2, χ_4

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Λ^- Spectrum: P-wave $\Sigma \pi \pi$ thresholds



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Expanding the Correlation Matrix via χ_1 , χ_2 , χ_4 N^+ Spectrum

Λ⁻ Spectrum: All multi-particle thresholds


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Low-lying Λ^- spectrum



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Low-lying Λ^- spectrum and S-wave thresholds



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Roper N1/2⁻ HSC Comparison A(1405)

Summary

- Several fermion-source and -sink smearing levels have been used to construct correlation matrices.
- A variety of 4 × 4, 8 × 8 and 12 × 12 matrices have been considered to explore the eigenstate energies revealed by different interpolating field structures.
- A low-lying Roper state has been identified in both quenched and full QCD using this correlation-matrix based method.
- The approach to the chiral limit is significantly different.
- The two heaviest quark masses considered in the dynamical case provide states consistent with πN multi-particle states.

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Roper N1/2⁻ HSC Comparison A(1405)

Summary continued...

- The *N*1/2⁻ results in quenched and dynamical QCD reveal significant differences in the approach to the physical point.
- A level crossing between the Roper and $N1/2^-$ states is observed in quenched QCD at $m_{\pi} \simeq 400$ MeV.
- A level crossing between the Roper and $N1/2^-$ states is anticipated in full QCD at $m_{\pi} \simeq 150$ MeV, just above the physical pion mass.
- The approach to the experimentally measured masses in full QCD is encouraging.
- The effects of the finite volume on self-energy contributions and associated avoided level crossings remains to be resloved.

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Roper $N1/2^-$ HSC Comparison $\Lambda(1405)$

Hadron Spectrum Collaboration Results Comparison

- At the heaviest mass compared, we find the same number of N⁺ states and qualitative agreement with the spectral energies.
- Finite-volume shifting of the P-wave $N\pi$ threshold is apparent in the spectra.
- Low-lying multi-particle states are suppressed on our large volume lattice for the three lighest quark masses.
- Qualitative agreement of the remaining *N*⁺ states is manifest.
- Qualitative agreement is also observed for the lowest lying N⁻ states.
- Derivatives provided through the lower components of the Dirac spinors are sufficient to access the N¹/₂ specturm.

Roper N1/2⁻⁻ HSC Comparison A(1405)

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- At the lightest mass considered, the Λ^- energy is sitting between the $\Sigma \pi$ and *NK* thresholds as the $\Lambda(1405)$ does in Nature.
- While the lowest Λ⁻ is predominantly flavour-singlet, its energy remains high.
- Again, effects of the finite volume on self-energy contributions and associated avoided level crossings remains to be resolved.
- Future work will expand the study to a complete set of interpolators and explore the flavour mixings of the low-lying states.

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Variational Method Summary of Results Λ(1405)

∧[−] Spectrum



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